OU-TAP-53
January 1997

# STABILITY OF SCALE-INVARIANT COSMOLOGICAL CORRELATION FUNCTIONS IN THE STRONGLY NON-LINEAR CLUSTERING REGIME

Taihei Yano and Naoteru Gouda
Department of Earth and Space Science, Graduate School of Science, Osaka University
Toyonaka, Osaka 560, Japan
E-mail: yano, gouda@vega.ess.sci.osaka-u.ac.jp

## ABSTRACT

We investigate stability of the scale-invariant solutions of the BBGKY equations for two-point spatial correlation functions of the density fluctuations in the strongly non-linear regime. In the case that the background skewness of the velocity field is equal to 0, we found that there is no local instability in the strongly non-linear regime. The perturbation does not grow nor does it decay. It has an only marginal stable mode. This result means that no special value of the power index of the two-point spatial correlation function are favored in terms of the stability of the solutions. In other words, the argument about the stability does not determine the power index of the two-point spatial correlation functions in the strongly non-linear regime.

*Subject headings*: cosmology:theory-large scale structures-correlation function

## 1. INTRODUCTION

The large scale structure formation is one of the most important probrems in the cosmology. It is generally believed that these structures have been formed due to gravitational instability. Hence it is very important to clarify the evolution of density fluctuations by gravitational instability. Here we consider the density fluctuations of the collisionless particles such as dark matters because our interest is concentrated on the effect of the self-gravity.

When the density fluctuations are much smaller than unity, that is, in the linear regime, time evolutions of the small density fluctuations can be analyzed by making use of the linear theory. In this regime, we can understand analytically how the small fluctuations grow(Peebles 1980, 1993). But when the density fluctuations are much larger than unity, that is, in the strongly nonlinear regime, the analytical approach is very difficult. But we believe that it is very necessary to understand clearly the nonlinear behavior of the density fluctuations on these strongly nonlinear scales because galaxy formations are much related to the density fluctuation on these small scales and also it is a very interesting academic problem for the nonlinear dynamics of the self-gravity.

As one way to quantify the clustering pattern, we generally use a two-point spatial correlation function of density fluctuations. In the strongly non-linear regime, it is found from $N$-body simulations that two-point spatial correlation functions obey the power law. This result is reasonable because the self-gravity is scale-free. Two-point spatial correlation function has been investigated by various methods because the power index of the two-point spatial correlation function is a good indicator representing the nonlinear dynamics of the self-gravity in this regime.

The power index of the two-point spatial correlation function have been usually analyzed by $N$-body simulations(Frenk, White, & Davis 1983; Davis et.al. 1985; Suto 1993 and references therein). However, the physical process that determine the value of the power index cannot be clarrified only by the $N$-body simulations. There are some other methods besides this numerical simulation. One of them is the analysis by the BBGKY equations. The work by Davis & Peebles (1977; hereafter DP) is a pioneer for the analysis by the BBGKY equations. They showed the existence of the self-similar solutions for spatial correlation functions under some assumptions. Then it is shown that the power index $\gamma$ of the two-point spatial correlation function in the strongly nonlinear regime is related to the initial power index $n$ of the initial power spectrum $P(k)$ as follows;

$$\xi(r) \propto r^{-\gamma}(\xi \gg 1 \ : \ \gamma = \frac{3(3+n)}{5+n}) \tag{1}$$

One of the assumptions that DP adopted is called the stable condition. This condition is that the mean relative physical velocity in the strongly nonlinear regime is equal to zero. This condition was tested by the $N$-body simulations (Efstathiou et al. 1988; Jain 1995), but this condition is not completely verified. Furthermore the stability of the scale-invariant solutions in the strongly nonlinear regime, which DP derived, was investigated by the linear perturbation theory(Ruamsuwan & Fry 1992;hereafter RF) because there is no guarantee that such solutions can exist stably. And it is found that the solutions are marginally stable.

As for the physical process determining the power index in the strongly nonlinear regime, there are other analysis besides one proposed by DP. One of them is given by Saslaw(1980). He concluded that the power index $\gamma$ approaches to 2 by using the cosmic energy equation under some assumptions while some numerical simulations do not support this result (Frenk, White & Davis 1983; Davis et al. 1985; Fry & Melott 1985).

There is another idea as follows; when the initial power spectrum has the sharp cut-off or the initial power spectrum is scale-free with negative and small initial power index, then there appear anywhere caustics of the density fields. In these cases, the power index is irrespective of the detailed initial conditions after the first appearance of caustics on the small scales around the typical size of the thickness of caustics(in three-dimensional systems, they correspond to the pancake structures of highly clustered matters). The power index is determined by the type of these caustics which is classified in accord with the catastrophe theory. This idea is verified in the one-dimensional system (Kotok & Shandarin 1988; Gouda & Nakamura 1988, 1989), the spherically symmetric systems(Gouda 1989), the two-dimensional systems (Gouda 1996a) and also the three-dimensional systems(Gouda 1996b). In these cases, it is suggested that $\gamma \approx 0$ on the small scales. As we can see from the above arguments, we believe that there are still uncertainties about the physical processes which determine the value of the power index.

Yano & Gouda (1996;hereafter YG) investigated the conditions that determine the power index of the two-point spatial correlation function in the strongly nonlinear regime by analysing the scale-invariant solutions of the BBGKY equations in this regime. YG does not adopt the following assumptions that DP adopted; (1) the skewness is equal to 0, (2) three-point spatial correlation function is represented by a product of the two-point spatial correlation function, (3) the stable condition is satisfied. As a result, YG obtained the relation between a mean relative pecurier velocity, skewness, three-body correlation function and the power index of the two-point spatial correlation function. YG found that the stable condition is not independent one of the other assumptions (1) and (2). That is, the stable condition is satisfied only when both assumptions (1) and (2) are satisfied. The assumptions (1) and (2) cannot be generally satisfied in all cases and then there is no guarantee that the stable condition are correct. Furthermore YG suggests from the physical point of view the probable range of the mean peculier velocity which includes the the value given by the stable condition. This fact results in the possibility that the power index of the two-point spatial correlation functions takes various values according to the mean peculier velocity. Indeed, YG showed that the mean relative physical pecurier velocity can take the value between 0 (stable clustering) and the Hubble expansion velocity (comoving clustering). When the stable clustering picture and the self-similerity are satisfied, the power index of the two-point spatial correlation function have the value that DP derived. On the other

hand, when the comoving clustering picture and the self-similerity are satisfied, the power index of the two-point spatial correlation function have the value of 0. And this value is consistent with the result from the catastrophy theory(Gouda& Nakamura 1988 1989;Gouda 1989 1996a;b). Although we found that there exist various scale-invariant solutions with different power index, whether these solutions are stable or not is an another interesting probrem. Some values of the power index of the two point spatial correlation function may not be able to be taken in the real world if these solutions are unstable. RF investigated the stability of the DP solutions by making use of the linear perturbation. And they showed that the perturbations of the solutions is marginal stable. But they investigated only in the DP case and did not investigate the other solutions that YG obtained. Furthermore, as we will discuss later, RF mistook the way of providing the perturbation about the skewness although their result is correct fortunately due to the reason which we will show later (§3). Furthermore they did not comment about the 'strange' growing mode which exist in their solutions of the perturbation. We will show that this 'strange' modes are resulted from the fact that RF put the inapplicable form of the perturbation which diverges on small or large scales. So, in this paper, we investigate the stability of the general solutions that YG obtained by putting the applicable form of the perturbation. We will derive the perturbation equation by perturbed BBGKY equation from the background scale-invariant solutions in the strongly nonlinear regime. And we will show the solution of the linear perturbation equation when we assume the appropriate form of the perturbation which is well-defined.

In §2, we briefly show the BBGKY equations that we use in this paper and also show the scale-invariant solutions in the strongly non-linear regime that YG obtained. In §3, we consider the stability of these solutions of the BBGKY equations by the analysis of the linear perturbation. Finally, we devote §4 to conclusions and discussions.

## 2. BASIC EQUATIONS AND THE SCALE-INVARIANT SOLUTIONS

At first, we show the BBGKY equations that we use in this paper. These equations are time evolution equations of the statistical value such as two-body correlation function, three-body correlation function, and so on. These equations can be derived from the ensumble mean of the Vlasov equation (DP,RF). The $N$-th BBGKY equation represents the time evolution of the $N$-body correlation function. We are now interested in the two-body correlation function, and then, we use the second BBGKY equation by taking the momentum moment of this equation. Zeroth moment of the second BBGKY equation becomes the time evolution equation of the two-point spatial correlation function. This

equation involves the first moment term, that is, the term of the mean relative peculier velocity. The first moment of the second BBGKY equation becomes the time evolution equation of the mean relative peculier velocity. This equation involves the second moment term of the relative peculier velocity dispersion $\Pi$, $\Sigma$, where, $\Pi$ ,and $\Sigma$ are pararel and transverse component of the mean relative peculier velocity dispersion, respectively (DP,RF,YG). The second moment of the second BBGKY equation becomes the time evolution equations of the relative peculier velocity dispersion. These equations invole the skewness of the velocity field in the same way. In deriving the BBGKY equations, DP assumed that the skewness is equal to 0, and also the stable condition in which the mean relative peculier physical velocity is equal to 0 ($\langle v \rangle = -\dot{a}x$). Furthermore DP assumed that a three-point spatial correlation function can be represented by a product of the two-point spatial correlation function as follows;

$$\zeta_{123} = Q(\xi_{12}\xi_{23} + \xi_{23}\xi_{31} + \xi_{31}\xi_{12}). \tag{2}$$

On the other hand, RF incorporated the skewness in the equations. RF expressed the skewness by using the values $A$, and $B$. These are related to our form by the next relations.

$$A(3v\Pi_{RF} + v^3) = 3\langle v \rangle \Pi + s_{\parallel}, \qquad B v \Sigma = \langle v \rangle \Sigma + s_{\perp}, \tag{3}$$

where $v$ in RF is the same as $\langle v \rangle$ in our paper, and $\Sigma$ in RF is also the same as our $\Sigma$. The definition of $\Pi_{RF}$ is different from ours ($\Pi_{RF} + v^2 = \Pi$). Here $s_{\parallel}$, $s_{\perp}$ are, as we will define later, the parallel component and the transverse component of the skewness, respectively. When we investigate the scale-invariant solutions of the BBGKY equations, the difference of the definition of the valuables in RF form and ours is not important. But when we perturbe the each valuable, the skewness also must be perturbed independently. However it must be noted that RF treated $A$ and $B$ as constant values. Then RF did not correctly treat with the perturbation of the skewness. RF used the same assumption that DP used about the three-point spatial correlation function. Furthermore, they also used the stable condition. We do not know whether the assumption of the three-point spatial correlation function is correct or not. Then we assume the three-point spatial correlation function by the following form;

$$\zeta_{123} = Q(\xi_{12}^{1+\delta}\xi_{23}^{1+\delta} + \xi_{23}^{1+\delta}\xi_{31}^{1+\delta} + \xi_{31}^{1+\delta}\xi_{12}^{1+\delta}), \tag{4}$$

where $\delta$ is a constant value. This is not a general form, but an extention of the form that DP adopted. We use this form as a preliminary step for our analysis. Furthermore the stable condition is a special case of the mean relative peculier velocity. Hence we do not

assume the stable condition and consider the general condition as shown in YG. We are interested in the strongly nonlinear regime, and then we take the non-linear approximation. In the strongly nonlinear regime ( $x \ll 1$), the two-point spatial correlation function is much larger than unity, $\xi \gg 1$. In this limit, we obtain the following four equations (DP 1977;RF 1992;YG 1996);

$$\frac{\partial \xi}{\partial t} + \frac{1}{a}\frac{1}{x^2}\frac{\partial}{\partial x}[x^2 \xi \langle v \rangle] = 0, \qquad \text{(0th moment)} \tag{5}$$

$$\begin{aligned} \frac{1}{ax^2}\frac{\partial}{\partial x}(x^2 \xi \Pi) &- \frac{2\xi\Sigma}{ax} \\ &+ 2Gm\bar{n}aQ\frac{x^\beta}{x}\int \frac{x_{31}^\beta}{x_{31}^3}\{\xi(x)^{1+\delta} + \xi(z)^{1+\delta}\}\xi(z-x)^{1+\delta} d^3x_3 = 0, \\ & \qquad\qquad \text{(1st moment)} \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{1}{a^2}\frac{\partial}{\partial t}[a^2\xi\Pi] + \frac{1}{ax^2}\frac{\partial}{\partial x}\left[x^2\xi\{3\langle v\rangle\Pi + s_\parallel\}\right] - \frac{4\xi}{ax}\{\langle v\rangle\Sigma + s_\perp\} \\ + \ 4Gm\bar{n}aQ^*\frac{x^\beta x^\gamma}{x^2}\int \frac{x_{31}^\beta}{x_{31}^3}\{\xi(x)^{1+\delta} + \xi(z)^{1+\delta}\}\xi(z-x)^{1+\delta}\langle v^\gamma\rangle d^3x_3 = 0, \\ \text{(2nd moment : contraction 1)} \end{aligned} \tag{7}$$

$$\begin{aligned} \frac{1}{a^2}\frac{\partial}{\partial t}[a^2\xi\Sigma] \ + \ \frac{1}{a}\frac{1}{x^4}\frac{\partial}{\partial x}\left[x^4\xi\{\langle v\rangle\Sigma + s_\perp\}\right] = 0, \\ \text{(2nd moment : contraction 2)} \end{aligned} \tag{8}$$

where $G$ is the gravitational constant, $m$ is the mass of a particle, $\bar{n}$ is the mean number density of the particles, $Q^*$ is the coeficient of the first momentum moment of the three-body correlation function (RF,YG). As YG commented, the fourth term of eq.(7) is not satisfied in general although this term is correct in the strongly non-linear regime in general. However DP showed the existence of the three-body correlation function which gives this term. As commented later, the condition that $Q^* = Q$ is consistent with the zero skewness case [see eq.(42)]. Since we consider the case of the zero skewness, we assume the fourth term of eq.(7) is satisfied in our analysis.

We express the skewness by the following expression.

$$s^{\alpha\beta\gamma} \equiv \langle (v - \langle v\rangle)^\alpha (v - \langle v\rangle)^\beta (v - \langle v\rangle)^\gamma \rangle = s_\parallel \ P_{ppp}^{\alpha\beta\gamma} + s_\perp \ P_{ptt}^{\alpha\beta\gamma}, \tag{9}$$

$$P_{ppp}^{\alpha\beta\gamma} = \frac{x^\alpha x^\beta x^\gamma}{x^3}, \qquad P_{ptt}^{\alpha\beta\gamma} = \frac{x^\alpha}{x}\delta^{\beta\gamma} + \frac{x^\beta}{x}\delta^{\gamma\alpha} + \frac{x^\gamma}{x}\delta^{\alpha\beta} - 3\frac{x^\alpha x^\beta x^\gamma}{x^3}. \tag{10}$$

where the subscripts $p$ and $t$ represent the parallel and transverse component of each two particles, respectively. Here $P_{ppt}$ and $P_{ttt}$ vanish because of the symmetry of the background universe. We use the Einstein-de Sitter Universe through this paper because we are interested only in the scale-invariant correlations.

Here we consider the scale-invariant solutions of these equations. In the strongly nonlinear regime, it is naturally expected that the effect of the nonlinear gravitational clustering dominates and then the solutions in this regime have no characteristic scales, that is, they are expected to obey the power law due to the scale-free of the gravity. Then we investigate the power law solutions of the $\xi, \langle v \rangle, \Pi$ and $\Sigma$ (YG). We assume that the two-point spatial correlation function $\xi$ is given by

$$\xi = \xi_0 a^{\beta} x^{-\gamma}. \tag{11}$$

Then, we obtain from the dimensional analysis in eq.(5)

$$\begin{aligned} \langle v \rangle &= -h\dot{a}x, \\ \beta &= (3-\gamma)h. \end{aligned} \tag{12}$$

In this case, the solutions of the other valuables are given by

$$\begin{aligned} \Pi &= \Pi_0 a^{\beta(1+2\delta)-1} x^{2-\gamma(1+2\delta)}, \\ \Sigma &= \Sigma_0 a^{\beta(1+2\delta)-1} x^{2-\gamma(1+2\delta)}, \\ s_{\parallel} &= s_{\parallel 0} \dot{a} a^{\beta(1+2\delta)-1} x^{3-\gamma(1+2\delta)}, \\ s_{\perp} &= s_{\perp 0} \dot{a} a^{\beta(1+2\delta)-1} x^{3-\gamma(1+2\delta)}. \end{aligned} \tag{13}$$

From eq.(8), it is found that

$$2\beta(1+\delta) + 1 - \{7 - 2\gamma(1+\delta)\}(h - \Delta) = 0, \tag{14}$$

and then

$$h = \frac{1 + \{7 - 2\gamma(1+\delta)\}\Delta}{1 - 6\delta}, \tag{15}$$

$$\Delta \equiv \frac{s_{\perp 0}}{\Sigma_0}. \tag{16}$$

As we can see from eq.(15), the parameter $h$ can take various values according to the skewness $\Delta$, the power index $\gamma$ and $\delta$. Only when $\Delta = \delta = 0$ is satisfied, the stable condition ($h = 1$) is correct. If the similarity solutions exist, the power index of the two-point spatial correlation function can be represented by the following form (Padmanabhan 1995;YG 1996);

$$\gamma = \frac{3(3+n)h}{2+(3+n)h}. \tag{17}$$

Then, even when we assume that the self-similarity solutions exist, the power index of the two-point spatial correltion function can take various values according to the mean relative peculier velocity, $h$ (if the stable condition is satisfied, i.e., $h = 1$, the result of DP[eq.(1)] is reproduced).

### 3. LINEAR STABILITY OF SCALE-INVARIANT SOLUTION

In the previous section, we showed that there are various scale-invariant solutions in addition to the solutions that DP obtained (YG 1996). But there is no guarantee that all the solutions can exist stably. As RF did, we investigate the stability of the solutions in the strongly nonlinear regime by making use of the linear perturbations theory. Now we perturbe the two-point spatial correlation function from the scale-invariant solution that we obtained in the previous section as follows;

$$\xi' = \xi(1 + \Delta_\xi), \tag{18}$$

where $\xi$ is the scale-invariant solution, $\xi'$ is the perturbed one and $\Delta_\xi \ll 1$. We also perturbe the other valuables such as the mean relative pecurier velocity $\langle v \rangle$, the relative pecurier velocity dispersions $\Pi$, $\Sigma$ and the skewness $s_\parallel$, $s_\perp$ in the same way.

The equations of the linear perturbations are following;

$$\frac{\partial \xi \Delta_\xi}{\partial t} + \frac{1}{ax^2}\frac{\partial}{\partial x}[x^2 \xi \langle v \rangle \{\Delta_\xi + \Delta_{\langle v \rangle}\}] = 0, \qquad \text{(0th moment)} \tag{19}$$

$$\begin{aligned} \frac{1}{ax^2}\frac{\partial}{\partial x}[x^2 \xi \Pi \{\Delta_\xi + \Delta_\Pi\}] \quad &- \quad \frac{2x\xi\Sigma}{ax}\{\Delta_\xi + \Delta_\Sigma\} \\ &+ \quad 2Gm\bar{n}aQx\xi^{2(1+\delta)}(1+\delta)M'_{\gamma(1+\delta),q}\Delta_\xi = 0, \\ &\qquad\qquad \text{(1st moment)} \end{aligned} \tag{20}$$

$$\begin{aligned}
&\frac{1}{a^2}\frac{\partial}{\partial t}[a^2\xi\Pi]\{\Delta_\xi+\Delta_\Pi\} \\
&\qquad+\frac{1}{ax^2}\frac{\partial}{\partial x}\left[x^2\xi\{3\langle v\rangle\Pi\{\Delta_\xi+\Delta_{\langle v\rangle}+\Delta_\Pi\}+s_\parallel\{\Delta_\xi+\Delta_{s_\parallel}\}\}\right] \\
&\qquad-\frac{4\xi}{ax}\{\langle v\rangle\Sigma\{\Delta_\xi+\Delta_{\langle v\rangle}+\Delta_\Sigma\}+s_\perp\{\Delta_\xi+\Delta_{s_\perp}\}\} \\
&\qquad+4Gm\bar{n}aQ^*x\langle v\rangle\xi^{2(1+\delta)}[(1+\delta)M'_{\gamma(1+\delta),q}\Delta_\xi+M_{\gamma(1+\delta)}\Delta_{\langle v\rangle}]=0, \\
&\qquad\qquad\qquad\qquad(\text{2nd moment : contraction 1})
\end{aligned} \tag{21}$$

$$\begin{aligned}
&\frac{1}{a^2}\frac{\partial}{\partial t}[a^2\xi\Sigma\{\Delta_\xi+\Delta_\Sigma\}] \\
&\quad+\quad\frac{1}{ax^4}\frac{\partial}{\partial x}\left[x^4\xi[\langle v\rangle\Sigma\{\Delta_\xi+\Delta_{\langle v\rangle}+\Delta_\Sigma\}+s_\perp\{\Delta_\xi+\Delta_{s_\perp}\}]\right]=0. \\
&\qquad\qquad\qquad\qquad(\text{2nd moment : contraction 2})
\end{aligned} \tag{22}$$

Here it should be noted that in deriving the above equations (19)-(22) we neglect some terms in full equations because of the following two reasons; one of them is that some terms are high order ones in the strongly non-linear limit. Another reason is that some terms are the higher order ones in the limit of small perturbation (larger than the first order perturbation). he ordering parameters in bothlimits are generally independent of each other. However we consider the case that the higher order terms in the strongly non-linear limit are much smaller than the first order terms of the perturbation. Then we can derive the above equations by neglecting higher order terms in the nonlinear approximation. This case means that we neglect the effects of the higher order terms in the strongly non-linear approximation in considering the linear perturbation.

RF expected the power law perturbation given by the following;

$$\Delta_\xi=\epsilon_\xi a^p x^q. \tag{23}$$

In this case, $M_{\gamma(1+\delta)}$ and $M'_{\gamma(1+\delta),q}$ are given by

$$M_{\gamma(1+\delta)} = \int\frac{\mu}{y^2}s^{-\gamma(1+\delta)}(1+y^{-\gamma(1+\delta)})d^3y, \tag{24}$$

$$M'_{\gamma(1+\delta),q} = \int\frac{\mu}{y^2}[s^{q-\gamma(1+\delta)}(1+y^{-\gamma(1+\delta)})+s^{-\gamma(1+\delta)}(1+y^{q-\gamma(1+\delta)})]d^3y, \tag{25}$$

$$\mu = \frac{x^\alpha}{x}\frac{z^\alpha}{z},\quad y=\frac{x_{31}}{x_{21}}=\frac{z}{x},\quad s=\frac{x_{23}}{x_{21}}=(1+y^2-2y\mu)^{1/2}. \tag{26}$$

The integrals should not diverge. Then, $2-\gamma(1+\delta)>0$ must be satisfied for $y\to 0$ and $\gamma(1+\delta)>0$ for $y\to\infty$ in the $M$. Furthermore, $2+q-\gamma(1+\delta)>0$ must be satisfied

for $y \to 0$ and $q - \gamma(1+\delta) < 0$ for $y \to \infty$ in the $M'$. As a result, the following relations must be satisfied;

$$0 < \gamma(1+\delta) < 2, \qquad \gamma(1+\delta) - 2 < q < \gamma(1+\delta). \tag{27}$$

The four perturbation equations (19)-(22) are a little different from the equations that RF derived. This is because in perturbating the three-body correlation term, RF devide the $\langle v_{21} \rangle$ into $\langle v_{23} \rangle + \langle v_{31} \rangle$ artificially and perturbed $\langle v_{23} \rangle$ and $\langle v_{31} \rangle$ independently, which is incorrect treatment. Furthermore, although they obtained the $q$-dependent of $M'$, they used the value of $M'$ only at $q = 0$ in solving the perturbation equations.

Here we consider the following form of the perturbation. We need to put the perturbations that do not diverge in the strongly non-linear limit ($x \to 0$) and the linear limit ($x \to \infty$). Then we put the perturbation of the two-point spatial correlation function as follows;

$$\Delta_\xi = \epsilon_\xi a^p x^q, \tag{28}$$

$$q = |q| i, \tag{29}$$

where $|q|$ is the real number, that is, $q$ is the pure imaginary number while RF adopted the real number. In this case, the perturbations do not diverge in the strongly non-linear limit and also in the linear limit. If $q$ is real number as RF adopted, the perturbation diverges on some scales. When $q$ is negative, the perturbation diverges in the non-linear limit ($x \to 0$). On the other hand, when $q$ is positive, the perturbation diverges in the linear limit ($x \to \infty$). The perturbations of the other valuables also can be put in the same form. From the dimensional analysis, all perturbations must have the same value of the power ($q$ and $p$) as that of the two-point spatial correlation function. When $q$ is pure imaginary number, $|s^q| = 1$ and $|y^q| = 1$. And the following relations are satisfied;

$$|M'_{\gamma(1+\delta),q}| \leq M'_{\gamma(1+\delta),q=0} = 2M_{\gamma(1+\delta)}. \tag{30}$$

When $0 < \gamma(1+\delta) < 2$, both $M_{\gamma(1+\delta)}$ and $M'_{\gamma(1+\delta),q}$ are finite for any value of pure imaginary number $q$.

These perturbed BBGKY equations are not closed by themselves in general and higher moment equations are needed. But if the coefficient of the perturbation of the skewness ($\Delta_{s_\parallel}$ and $\Delta_{s_\perp}$) are equal to 0, the perturbation equations for the other valuables have no relation to the perturbation of the skewness. In this case we can solve these perturbation equations independently from the higher moment equations. As we can see from eqs.(21) and (22), when the skewness of the background velocity field is equal to 0, the coefficient

of the perturbation of the skewness becomes 0. Here we consider only this case that the skewness is equal to 0. Then we can neglect the higher moment equations. RF treated $A$ and $B$ as a constant value. So, the treatment of the perturbation of the skewness itself was mistaken. But when the skewness is equal to 0, RF treatment happen to be correct fortunately. In this case, the above equations are rewriten by putting the perturbations which have the power law given by eqs.(28) and (29) as follows;

$$(p - hq)\Delta_\xi - h(3 - \gamma + q)\Delta_{\langle v\rangle} = 0, \tag{31}$$

$$\begin{aligned}&[2\{\gamma(1+\delta) - 2 + \sigma\}(1+2\delta) + q + 2(1+\delta)Dkq]\Delta_\xi \\ + \quad &\{4 - 2\gamma(1+\delta) + q\}\Delta_\Pi - 2\sigma\Delta_\Sigma = 0,\end{aligned} \tag{32}$$

$$\begin{aligned}&[1 - 15h + (6+4\gamma)h(1+\delta) + 4h\sigma + p - 3hq \\ &\quad -8h\frac{Q^*}{Q}\{\gamma(1+\delta) - 2 + \sigma\}(1+\delta) - 4h\frac{Q^*}{Q}(1+\delta)Dkq]\Delta_\xi \\ &\quad +[-3h\{5 - 2\gamma(1+\delta) + q\} + 4h\sigma - 4h\frac{Q^*}{Q}\{\gamma(1+\delta) - 2 + \sigma\}]\Delta_{\langle v\rangle} \\ &\quad +[1 - 15h + (6+4\gamma)h(1+\delta) + p - 3hq]\Delta_\Pi + 4h\sigma\Delta_\Sigma = 0,\end{aligned} \tag{33}$$

$$(1 - h + 6h\delta + p - hq)\Delta_\xi - h\{7 - 2\gamma(1+\delta) + q\}\Delta_{\langle v\rangle} + (1 - h + 6h\delta + p - hq)\Delta_\Sigma = 0, \tag{34}$$

where

$$D \equiv \gamma(1+\delta) - 2 + \sigma, \qquad \sigma = \frac{\Sigma}{\Pi}. \tag{35}$$

It is difficult to treat exactly $M'$ as a function of $q$. We approximate $M'$ by linear function of $q$. When $q = 0$, $M'/M$ is equal to 2. Then we use the following approximation of $M'/M$ in the above equations (20) and (21);

$$\frac{M'}{M} \equiv 2 + kq, \tag{36}$$

where

$$k = \frac{\int \frac{\mu}{y^2}[s^{-\gamma(1+\delta)}(1+y^{-\gamma(1+\delta)})\log s + s^{-\gamma(1+\delta)}y^{-\gamma(1+\delta)}\log y]d^3y}{\int \frac{\mu}{y^2}s^{-\gamma(1+\delta)}(1+y^{-\gamma(1+\delta)})d^3y}. \quad (37)$$

The integral $\int(\mu/y^2)s^{-\gamma(1+\delta)}(1+y^{-\gamma(1+\delta)})d^3y$ is dominated arround the $s$ and $y \sim 1$. Arround the $s$ and $y \sim 1$, $\log s$ and $\log y$ have the values of order 1. So, $k$ has the value of order 1.

Furthermore we use the following relation that can be derived from the first moment equation [eq.(6)];

$$4 - 2\gamma(1+\delta) - 2\sigma + 2Gm\bar{n}a^2Qx^2\frac{\xi^{1+2\delta}}{\Pi}M_{\gamma(1+\delta)} = 0. \quad (38)$$

These four equations [eqs.(31)-(34)] can be rewriten by using a matrix expression.

$$N_{ij}u_j = 0, \quad (39)$$

$$u_j = (\Delta_\xi, \Delta_{\langle v\rangle}, \Delta_\Pi, \Delta_\Sigma). \quad (40)$$

If there exist a non trivial solution, the determinant of the matrix $N_{ij}$ should be equal to 0. Now we consider the zero skewness case, there is the relation between the three-point spatial correlation function and the mean relative pecurier velocity by using eq.(15).

$$\delta = \frac{h-1}{6h}. \quad (41)$$

By using this relation, we can eliminate $\delta$ in eqs.(32)-(34). Furthermore, from the first moment equation(6) and the second moment (contraction 1) equation (7), we obtain the following relation;

$$(-h+1+6h\delta)\Pi + \{5-2\gamma(1+\delta)\}\frac{s_\parallel}{\dot{a}x} - \frac{4s_\perp}{\dot{a}x} - 4Gm\bar{n}a^2x^2M_{\gamma(1+\delta)}h(Q^*-Q) = 0. \quad (42)$$

As we can see, when the skewness is equal to 0, $Q^*-Q$ must be 0, that is, $Q^*/Q$ is equal to 1.

Then the determinant of $\boldsymbol{N}$ is given by

$$\det \boldsymbol{N} = \frac{1}{3}(p-hq)^2 f(q,\gamma',h,\sigma,kD), \quad (43)$$

where

$$
\begin{aligned}
f(q,\gamma',h,\sigma,kD) &= \{9h + kD(7h-1)\}q^2 \\
&+ (4 - 2\gamma' + 29h - 10\gamma' h - 2\sigma + 8h\sigma)q \\
&+ kD(-3 + 21h - 6\gamma' h)q + 12 - 6\gamma' - 6\sigma,
\end{aligned} \tag{44}
$$

and

$$\gamma' \equiv \gamma(1+\delta). \tag{45}$$

Here $f$ is the quadratic equation of $q$. Now we investigate whether the equation $f = 0$ has real solutions or not. Since we do not know the value $\gamma'$, $h$, $\sigma$, and $kD$ in the strongly non-linear regime, we treat these values as parameters. Here we consider the allowed range of these parameter. As we can see from eq.(27), the parameter $\gamma'$ must be satisfied $0 < \gamma' < 2$. Since we have treated as $\zeta \gg \xi$ in the strongly non-linear regime, $\xi^{2(1+\delta)}$ should be higher order than $\xi$. This results in that $2(1+\delta) > 1$ must be satisfied. In this case, $h > \frac{1}{4}$ must be satisfied from eq.(41). YG showed the probable range of the mean relative peculier velocity and obtained that the mean relative physical pecurier velocity must have the value between 0 and the Hubble expansion velocity. This means that the parameter $h$(relative velocity parameter) have the value between 0 and 1. So in this case, the parameter $h$ should be in the range $\frac{1}{4} < h < 1$. We do not know the range about the parameters $\sigma$ and $kD$. But the parameter $\sigma$ sould have an order of 1. Hence we investigate $\sigma$ in the range $\frac{1}{2} < \sigma < 2$. The parameter $D$ and $k$ also take an order of 1 as seen from the eqs. (35) and (37), respectively. So we investigate $kD$ in the range $-1 < kD < 1$.

In the above probable value of the parameters, we can easily ascertain that $f = 0$ has real solutions. In other words, $f = 0$ does not have the solutions of imaginaly number. Since we consider the case that $q$ is imaginaly number, $f = 0$ can not be satisfied. $p - hq$ should be 0 to satisfy that the determinant of the matrix $\boldsymbol{N}$ is equal to 0. In this case, $p$ has the following value

$$p = hq = h|q|i. \tag{46}$$

This means that the perturbations do not grow. And the solutions are stable.

Furthermore we consider the strict condition which determines the stability of the two-point spatial correlation function. In the strongly non-linear regime, the mean relative peculier velocity has the value, $\langle v \rangle = -h\dot{a}x$ according to the process of clustering. In this case, the scale of $a^h x$ for the two particles whose mean comoving distance is $x$ does not change as we can see from the following relation;

$$\frac{d}{dt}(a^h x) \;=\; a^{h-1}(a\dot{x} + h\dot{a}x)$$

$$
\begin{aligned}
&= a^{h-1}(\langle v\rangle + h\dot{a}x) \\
&= 0 \qquad (47)
\end{aligned}
$$

Then we should determine the stability of the two-point spatial correlation function at the fixed scale of $a^h x$. The solutions of the perturbation are rewritten by

$$
\begin{aligned}
\Delta_\xi &= \epsilon_\xi (a^h x)^q \\
&= \epsilon_\xi e^{i|q| log(a^h x)} \qquad (48)
\end{aligned}
$$

This perturbation does not grow nor does it decay. At the fixed scale of $a^h x$, the perturbation never even oscillate. This means that the perturbation is marginal stable. RF used the real number $q$ in investigating the behaviour of the perturbation. As we can see from eq.(48), the perturbation in the $p - hq = 0$ mode works well even when $q$ is real number. That is, the perturbation is marginal stable in this mode. However there also exist a 'strange' growing mode for the real number of $q$ because $f = 0$ is satisfied in this case. RF did not comment sufficiently about this 'strange' growing mode.

When $q$ is negative, the perturbation diverges in the non-linear limit ($x \rightarrow 0$). On the other hand, when $q$ is positive, the perturbation diverges in the linear limit ($x \rightarrow \infty$). Then this perturbation is not adequate in investigating the local stability of the two-point spatial correlation function in the strongly nonlinear regime.

## 4. RESULTS AND DISCUSSION

In this paper, we investigated the stability of the scale-invariant solutions of the cosmological BBGKY equations in the strongly nonlinear regime in the case that the skewness of the velocity field is equal to zero. The reason why we consider the only case that the skewness vanishes is that perturbed BBGKY equations for $\xi$, $\langle v\rangle$, $\Pi$ and $\Sigma$ can be closed independently of the higher moment perturbations. When the power law perturbations are put in the solutions as RF put, that is, when $q$ is real number, the perturbation in the linear limit or the non-linear limit diverges. When $q$ is negative, the perturbations diverge and do not work in the non-linear limit. So $q$ should be positive. On the other hand, when $q$ is positive, the perturbations work well in the non-linear regime. In the linear regime, however, those perturbations may diverge if the power law form of the pertubations are retained and so the form of the perturbations should be changed in order not to diverge in the linear limit. At this case, it is insufficient to solve the non-linear approximated equations because the informations about the evolutions on all scales are needed. Then we investigated only the local stability of the non-linear regime. In investigating the local

stability of the two-point spatial correlation function in the strongly non-linear regime, we should put the perturbation whose value is zero or smaller value on the scales except for the strongly non-linear scales than that on the strongly non-linear scalesthe investigating scale. That is, we should put the wave packet-like perturbation. In order to put such a wave packet-like perturbation, $q$ must be imaginaly number. In this case, we found that there is no unstable mode in the strongly non-linear regime. It seems stable for any value of the power index of the two-point spatial correlation function in the strongly non-linear regime. However we do not know whether a grobal unstability exist or not because we consider only the local stability. It is certain that there is no local unstability in the strongly non-linear regime. So, in the strongly non-linear regime, the solutions is marginally stable, and it does not seem that the power index of the two-point spatial correlation function approaches to some stable point values. The power index of the two-point spatial correlation function that was derived by DP, $\gamma = 3(3+n)/(5+n)$, is not the special one also in terms of the stability of the solution. As a result, the argument of the stability does not determine the power index of the two-point spatial correlation function in the strongly non-linear regime.

The power index of the two-point spatial correlation function is determined only by the clustering process, that is, the parameter $h$ if the self-similar solutions exist. Hence it is very important to estimate the parameter $h$ and investigate whether the self-similar solutions exist or not in the general scale-invariant solutions which YG obtained.

We are grateful to M.Nagashima for useful discussions. We would like to thank T.Tanaka for important comments. And we would like to thank S.Ikeuchi and M.Sasaki for useful suggestions and continuous encouragement. This work was supported in part by the Grant-in-Aid No.06640352 for the Scientific Research Fund from the Ministry of Education, Science and Culture of Japan.

## REFERENCES

Davis, M., Efstathiou, G., Frenk, C.S.,& White, S.D.M. 1985, ApJ, 292, 371

Davis, M., & Peebles, P.J.E. 1977, ApJS, 34, 425 (DP)

Efstathiou, G., Frenk, C.S., White, S.D.M., & Davis, M. 1988, MNRAS, 235, 715

Frenk, C.S., White, S.D.M., & Davis, M. 1983, ApJ, 271, 417

Fry, J.N. & Melott, A.L. 1985, ApJ, 292, 395

Gouda, N. 1989, Prog.Theor.Phys., 81, 648

Gouda, N.1996a, ApJ, 469, 455

Gouda, N.1996b, preprint (OU-TAP 47)

Gouda, N., & Nakamura, T. 1988, Prog.Theor.Phys., 79, 765

Gouda, N., & Nakamura, T. 1989, Prog.Theor.Phys., 81, 633

Jain, B. 1995, preprint(astro-ph/9509033)

Kotok, E. & Shandarin, S.F. 1988, Soviet Astr., 32, 351

Padmanabhan, T. 1996, MNRAS, 278, L29

Peebles, P.J.E. 1980, The Large-Scale Structure of the Universe (Princeton:Princeton Univ.Press)

Peebles, P.J.E. 1993, The Principles of Physical Cosmology (Princeton:Princeton Univ.Press)

Ruamsuwan, L. & Fry, J.N. 1992, ApJ, 396, 416 (RF)

Saslaw, W.C. 1980, ApJ, 235, 299

Suto, Y. 1993, Prog.Theor.Phys., 90, 1173

Yano, T. & Gouda, N. 1996, preprint(astro-ph/9605032)(YG)

---